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Solitary waves with friction in one-dimensional anisotropic magnets

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Abstract. We study the coupling of solitary solutions of a magnetic Hamiltonian with strong easy-plane anisotropy to longitudinal phonon excitations of the surrounding medium. The spectral density of the coupling describing frequency-dependent damping is calculated. The spin-correlation functions and the partition function of the dissipative magnetic system are computed.

1. Introduction

In the past, several attempts have been made to introduce dissipation into magnetic systems described by the Heisenberg Hamiltonian and its 'relatives'. Among these are two very important subjects. The first is the sine–Gordon system, of which a good overview has been given by Bishop *et al* [1], and the second is the field of single-domain particles [2]. Most of the results concerned with these topics, including environmental effects, are collected together in a review by Stamp *et al* [3]. The present paper is concerned with the dissipative sine–Gordon system. The influence of dissipation has been discussed previously within the problem of single-domain particles.

Garg and Kim [4] assumed that the magnetization of a single-domain particle is coupled to a bath of phonons via the magnetoelastic tensor. In what follows, we will provide a generalization of their approach to the sine–Gordon field problem. A more phenomenological approach was followed by Loss *et al* [5], who described the dissipative motion of the azimuthal angle Φ of the total magnetization via a bilinear coupling of $\dot{\Phi}$ to the phonon coordinate.

In the sine–Gordon problem also, several dissipative models were established. In some of the studies [6], a phenomenological system-plus-reservoir model [7] with a bilinear coupling in the sine–Gordon field $\Phi(x,\tau)$ and in the environmental coordinates has been studied. Hänggi *et al* [8] supplemented the classical equation of motion with a phenomenological damping term $-\eta\dot{\Phi}$. The same *ansatz* was used by McLaughlin and Overman [9], who studied the thermal decay of breather solutions of the sine–Gordon equation. A slightly different damping term, $\alpha\dot{\Phi} - \beta\dot{\Phi}''$, was used by Salerno *et al* [10]. Clearly, in order to justify the phenomenological approaches microscopically, one has to investigate the coupling of the magnetic degrees of freedom to the elementary excitations of the surrounding medium. Wada and co-workers [11] examined the nonlinear interaction between solitary excitations and linear oscillating modes of the sine–Gordon equation, and, as a result of the coupling, they found diffusive motion of the former. Quite

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similarly, Rostiashvili and Schilling [12] showed that the coupling of domain walls to spatial fluctuations of the magnetization leads to an effective kink–kink interaction. Here we investigate the coupling of the sine–Gordon field to the surrounding lattice vibrations.

In section 2 we introduce the Lagrangian of the sine–Gordon system with the coupling to a bath of phonons. In section 3, the phonons are eliminated from the description, and the dynamics of the sine–Gordon field Φ is reduced to that of a collective coordinate. The partition function and the spin-correlation functions are calculated using the path-integral formulation. The spectral density of the coupling is studied in some detail in section 4. Before summarizing our work in section 6, we discuss in section 5 results for the change of the partition function and spin-correlation functions under the influence of the phonon coupling.

2. System-plus-reservoir models

The Lagrangian of an anisotropic magnetic system interacting with a thermal reservoir of phonons may be written as

$$L = L_S + L_{ph} + L_{int} + L_{CT} \tag{1}$$

where L_s describes the magnetic system, L_{ph} the thermal bath of phonons, and L_{int} is the interaction between the spins and the phonons. Under the assumption that any one phonon degree of freedom is only weakly perturbed, the interaction may be chosen to be linear in the phonon variables. However, L_{int} does not need to be linear in the magnetization. The last part of the Lagrangian is a counter-term which has been included to compensate potential renormalization effects due to the coupling [7].

The Lagrangian of the unperturbed one-dimensional spin system (directed along the z-axis) can be taken as the Lagrangian of the sine–Gordon problem which is deduced in the case of strong easy-plane anisotropy [13] from the continuum limit of the ferromagnetic Heisenberg Hamiltonian:

$$\hat{H}_H = -\sum_{\langle i,j\rangle} J\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j + A\sum_i (\hat{S}_i^z)^2 - g\mu_B B\sum_i \hat{S}_i^x. \tag{2}$$

Here J is the exchange integral, A is the anisotropy energy ($A > 0$), and B is an externally applied magnetic field directed along the x-axis. The sum $\sum_{\langle i,j\rangle}$ runs over pairs of nearest neighbours only. To perform the continuum limit, it is convenient to introduce a basis of coherent spin states $|\boldsymbol{\Omega}\rangle$, satisfying $\boldsymbol{\Omega}\cdot\hat{\boldsymbol{S}}|\boldsymbol{\Omega}\rangle = S|\boldsymbol{\Omega}\rangle$, where S is the spin eigenvalue and $\boldsymbol{\Omega} \equiv (\sin\theta\cos\Phi, \sin\theta\sin\Phi, \cos\theta)$ is a vector on the unit sphere associated with the spin direction. The partition function of the spin system may be written as a path integral over the spin configurations $\boldsymbol{\Omega}(z,\tau)$. In the limit of strong anisotropy, the path integral is dominated by the stationary solution $\theta(z,\tau) = \pi/2$. Within the Gaussian approximation for the fluctuations about these paths, the integration over θ can easily be performed. This procedure results in an effective Euclidean Lagrangian depending only on the angle Φ between the spin and the x-axis. In the limit of vanishing lattice spacing, the action takes the continuous form

$$L_S[\Phi(z,\tau)] = E_0 a\int \mathrm{d}z\,\left\{\frac{1}{2c^2}\dot{\Phi}^2 + \frac{1}{2}\left(\frac{\partial\Phi}{\partial z}\right)^2 + \frac{\omega_0^2}{c^2}(1-\cos(\Phi))\right\} \tag{3}$$

where $c = (2a^2JAS^2)^{1/2}$ and $\omega_0 = (2g\mu_B BS)^{1/2}$ are the characteristic velocity and frequency of the sine–Gordon problem, respectively. The energy scale $E_0 = JS^2$ is chosen in accordance with the Heisenberg model, and a is the lattice spacing along the z-axis.

The phonon part of the Lagrangian describing a continuous set of 1D phonons propagating along the chain reads

$$L_{ph}[U_\lambda(z,\tau)] = \rho_{Res}\sum_\lambda \int \mathrm{d}z\ \left\{\frac{1}{2}\dot{U}_\lambda^2 + \frac{v_\lambda^2}{2}\left(\frac{\partial U_\lambda}{\partial z}\right)^2\right\} \tag{4}$$

where ρ_{Res} is the mass density of the crystal, and $U_\lambda(z,\tau)$ is the λ- ($= x, y, z$-) component of the phonon field $\boldsymbol{U}(z,\tau)$.

We introduce the interaction between the magnetic system described by the Heisenberg Hamiltonian (2) and lattice vibrations (4) by allowing the exchange integral J to vary with the displacement of the relative position of nearest-neighbour sites carrying the spins. Since the variation might be different for different components of $\boldsymbol{S}$ (because of the anisotropy of the underlying solid), the exchange integral J has to be regarded as a tensor:

$$J \to J_{\mu\nu}(\boldsymbol{R}_i - \boldsymbol{R}_j) \approx J\delta_{\mu\nu} + (u_i^\lambda - u_j^\lambda)\left.\frac{\partial J_{\mu\nu}}{\partial x_i^\lambda}\right|_{\substack{\boldsymbol{R}_i=\boldsymbol{R}_i^0\\ \boldsymbol{R}_j=\boldsymbol{R}_j^0}} \tag{5}$$

where $u_i^\lambda = x_i^\lambda - z_i^0\delta_{\lambda z}$ is the displacement of ith spin from the equilibrium position. In the continuum limit, this will be replaced by the phonon field $U_\lambda(z,\tau)$ according to

$$J_{\mu\nu}(z_i - z_j)S_i^\mu S_j^\nu \approx J(z_i^0 - z_j^0)\delta_{\mu\nu}S_i^\mu S_j^\mu + \frac{\partial U_\lambda(z,\tau)}{\partial z}D_{\mu\nu}^{\lambda z}S_i^\mu S_j^\nu \tag{6}$$

where we have introduced the magnetoelastic tensor $D_{\mu\nu}^{\lambda z} = a(\partial/\partial x^\lambda)J_{\mu\nu}(a)$.

In the case of an exchange interaction depending only on the distance $|\boldsymbol{R}_i - \boldsymbol{R}_j|$, the quantity $\partial J_{\mu\nu}(|\boldsymbol{R}|)/\partial x^\lambda$ is different from zero only for $\lambda = z$, which means that only longitudinal phonons are coupled to the magnetic chain. For this reason, from now on we shall consider only longitudinal phonons and, for notational simplicity, we shall label the magnetoelastic tensor only with indices of the components in spin direction.

Taking the continuum limit for the spin product in (6):

$$S_i^\mu S_{i+1}^\nu \to S^\mu(z)S^\nu(z) + S^\mu(z)a\frac{\partial}{\partial z}S^\nu(z) \tag{7}$$

we finally obtain an interaction energy which is linear in the phonon field. Neglecting terms of order a^2U_λ, we have

$$H_{int} = \int_{L_z}\frac{\mathrm{d}z}{a}D_{\mu\nu}\frac{\partial U_\lambda(z)}{\partial z}S^\mu(z)S^\nu(z). \tag{8}$$

With the Lagrangian (1), the canonical density matrix may be written as the path integral expression [7]

$$\rho[\Phi_f, U_f, \beta; \Phi_i, U_i, 0] = \frac{1}{Z}\int_{\Phi(z,0)=\Phi_i}^{\Phi(z,\beta)=\Phi_f}\mathcal{D}\Phi\int_{U_\lambda(z,0)=U_i}^{U_\lambda(z,\beta)=U_f}\mathcal{D}U_\lambda\exp\{-S[\Phi(z,\tau), U_\lambda(z,\tau)]\} \tag{9}$$

where $\beta = 1/T$ is the 'thermal' time ($\hbar = k_B = 1$), and S is the Euclidean action

$$S[\Phi, U] = \int_0^\beta \mathrm{d}\tau\ L[\Phi(z,\tau), U_\lambda(z,\tau)]. \tag{10}$$

Furthermore, $Z = \mathrm{tr}\,\rho$ is the partition function of the spin-plus-reservoir system. The *reduced* density matrix, which acts only on the subspace of Φ, is calculated by performing the trace over the coordinates U_λ of the reservoir. It may be written as

$$\rho_{Red}[\Phi_f, \beta; \Phi_i, 0] = \frac{1}{Z_{Red}}\int_{\Phi(z,0)=\Phi_i}^{\Phi(z,\beta)=\Phi_f}\mathcal{D}\Phi\exp\{-S_S[\Phi(z,\tau)]\}\mathcal{F}_{inf}[\Phi(z,\tau)]. \tag{11}$$

The influence functional describing the influence of the thermal bath on the reduced system is given by

$$\mathcal{F}_{inf}[\Phi(z,\tau)] \equiv \exp\{-S_{inf}[\Phi]\} = \exp\{-S_{CT}[\Phi]\}\frac{1}{Z_{ph}}\oint \mathcal{D}U_\lambda \exp\{-S_{ph}[U_\lambda] - S_{int}[\Phi, U_\lambda]\}. \tag{12}$$

Here the path sum is over all periodic paths of period β, $U_\lambda(z,0) = U_\lambda(z,\beta)$ and $Z_{red} \equiv Z/Z_{ph}$ is the partition function of the reduced system. Since the action inside the influence functional (12) is quadratic in the phonon fields $U_\lambda(z,\tau)$, the path summation can be carried out exactly by solving Gaussian integrals [14]. Thus we obtain the influence action

$$S_{inf}[\Phi] = -\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau' \int \mathrm{d}z \int \mathrm{d}z'\, \mathcal{K}(\tau-\tau', z-z') F_\lambda(z,\tau) F_\lambda(z',\tau'). \tag{13}$$

The damping kernel is given by

$$\mathcal{K}(\tau-\tau', z-z') = \frac{1}{\rho_{Res}}\sum_{k\lambda}\left\{\frac{\cosh(\Omega_{k\lambda}[\beta/2 - |\tau-\tau'|])}{8\Omega_{k\lambda}\sinh(\Omega_{k\lambda}\beta/2)} - \frac{1}{4\Omega_{k\lambda}^2}\delta(\tau-\tau')\right\}\frac{\mathrm{e}^{-\mathrm{i}k_z(z-z')}}{2\pi} \tag{14}$$

with the dispersion relation $\Omega_{k\lambda} = v_\lambda|k_z|$, and the coupling functions

$$F_\lambda(z,\tau) = -\frac{1}{a}D^{\lambda z}_{\mu\nu}\frac{\partial}{\partial z}S^\mu(z,\tau)S^\nu(z,\tau). \tag{15}$$

The coupling functions (15) result from the interaction Hamiltonian (8) via partial integration. The summation over k in the expression (14) for $\mathcal{K}$ may be carried out exactly. In section 4 we study the expression (14) in more detail in order to extract the form of the spectral density connected with this kind of coupling. Before doing so, we introduce in the next section the classical solutions of the sine–Gordon equation and the method of collective coordinates.

3. Collective coordinates

The calculation of the density matrix for the unperturbed spin system (see equation (11) with $\mathcal{F}_{inf}$ set to 1) described by the sine–Gordon Lagrangian (1) is usually performed within the semiclassical approximation, in which the fluctuating field Φ around the stationary paths of the action is expanded up to second order. The stationary paths are solitary waves (the so-called kinks and antikinks)

$$\Phi_{Sol}(z,\tau) = 4\arctan\left\{\exp\left[\pm\frac{\omega_0}{c}\gamma(z-v\tau)\right]\right\} \tag{16}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, and the sign + (−) refers to kinks (antikinks). Because of the translational invariance of the classical action associated with the solitary waves, the fluctuations about the stationary path have a zero mode which needs special treatment [15]. To handle the zero mode appropriately, it is convenient to perform a canonical tranformation to a new set of variables [16] $\{q(\tau), \xi_{k_z}(\tau)\}$, where the variable $q(\tau)$ represents the centre of the kink, usually referred to as the 'collective coordinate' [17], and where $\xi_{k_z}(\tau)$ is the amplitude of the fluctuation mode $\Psi_{k_z}(z)$. We then have

$$\Phi(z,\tau) = \Phi_{Sol}(z - q(\tau)) + \eta(z,\tau) \qquad \eta(z,\tau) = \frac{c}{\sqrt{E_0 a}}\sum_{k_z}\xi_{k_z}(\tau)\Psi_{k_z}(z - q(\tau)). \tag{17}$$

In (17), the zero mode $\Psi_0(z)$ is omitted, and its part is played by the collective coordinate $q(\tau)$.

In this paper, we restrict ourselves to the low-temperature regime in which the kinks are moving slowly so that the 'nonrelativistic' limit $v \ll c$ applies. Thus we may put $\gamma \approx 1$ in (16). On using the collective coordinate $q(\tau)$ instead of $v\tau$, the Lagrangian L_S of the system given in (3) takes the nonrelativistic form

$$L_S^{(sol)}[q(\tau)] = M_{Sol} + \frac{M_{Sol}}{2c^2}\dot{q}^2(\tau) \tag{18}$$

with the soliton mass $M_{Sol} = 8JS^2\omega_0 a/c^3$.

The partition function receives contributions from sectors of the Hilbert space representing n kinks and $\bar{n}$ antikinks. At low temperatures, the density of kinks and antikinks is very small, so they may be treated as a noninteracting gas. In fact, it is sufficient to consider the contributions Z_0 and Z_1 of the zero- and one-soliton sector, respectively. Then the partition function may be written in the form [18]

$$Z = Z_0 \exp(2L_z n_{Sol}) \tag{19}$$

where $n_{Sol} = Z_1/Z_0 L_z$ describes the soliton density.

In the soliton-free sector, we have harmonic magnetic fluctuations, and the partition function reads

$$Z_0 = \prod_{k_z=0}^{\infty} \frac{1}{2\sinh(\omega_{k_z}\beta/2)} \tag{20}$$

where $\omega_{k_z}^2 = \omega_0^2 + c^2k_z^2$ is the dispersion relation of magnetic fluctuations [15].

In the one-soliton sector, the transformation to the collective coordinate gives rise to a Lagrangian which is nonlinear in the fluctuations. According to [17], however, the nonlinear contributions can be summed, and they lead to a renormalized soliton mass at low temperature:

$$M'_{Sol} \approx M_{Sol}\left(1 + \frac{\pi}{16S}\sqrt{\frac{A}{J}}\right). \tag{21}$$

Now we are ready to separate the Lagrangian into a part describing harmonic fluctuations and a part describing the soliton as a free particle, $L^{(1)} = L_{Sol} + L_{Fluc}$:

$$L_{Sol}[q(\tau), \dot{q}(\tau)] = M'_{Sol}c^2 + \frac{\dot{q}^2(\tau)}{2M'_{Sol}} \tag{22}$$

$$L_{Fluc}[\xi_{k_z}(\tau), \dot{\xi}_{k_z}(\tau)] = \tfrac{1}{2}\sum_{k_z}(|\xi_{k_z}(\tau)|^2 + \omega_{k_z}^2|\dot{\xi}_{k_z}(\tau)|^2). \tag{23}$$

The dispersion relation of the harmonic modes is the same as in the zero-soliton sector, $\omega_{k_z}^2 = \omega_0^2 + c^2k_z^2$. As a result of the dynamical independence of L_{Sol} and L_{Fluc}, the partition function in the one-soliton sector is in the factorized form $Z^{(1)} = Z_{Sol}Z_{Fluc}$, with

$$Z_{Sol} = \mathrm{e}^{-M'_{Sol}c^2\beta}\sqrt{\frac{M'_{Sol}}{2\pi\beta}} \tag{24}$$

$$Z_{Fluc} = \prod_{k_z=0}^{\infty} \frac{1}{2\sinh(\omega_{k_z}\beta/2)}. \tag{25}$$

In the soliton-free sector, the fluctuations $\eta(z,\tau)$ are about $\Phi_0(z,\tau)\equiv 0$. Expanding in the fluctuations up to second order, we find for the correlation functions

$$C^{(0)}(z,\tau)\equiv\langle\cos(\eta(z,\tau))\cos(\eta(0,0))\rangle^{(0)}\approx 1-\langle\eta^2(0,0)\rangle^{(0)}=\text{constant} \tag{26}$$

$$S^{(0)}(z,\tau)\equiv\langle\sin(\eta(z,\tau))\sin(\eta(0,0))\rangle^{(0)}\approx\langle\eta(z,\tau)\eta(0,0)\rangle^{(0)}. \tag{27}$$

The transversal spin-correlation function $S^{(0)}(z,\tau)$ has two delta peaks in frequency space:

$$\tilde{S}^{(0)}(k,\omega)=\frac{c^2}{cE_0\omega_{k_z}}\frac{1}{1-\mathrm{e}^{-\omega\beta}}(\delta(\omega-\omega_{k_z})-\delta(\omega+\omega_{k_z})). \tag{28}$$

In the one-soliton sector, we may use again the above canonical transformation, the decomposition (17), and the subsequent harmonic approximation for the fluctuation modes. If the length L_z of the system is large compared to the extension c/ω_0 of the kink, the main influence of the kink on the fluctuation modes is a phase shift at the centre of the kink. Then we may use the eigenfunctions of the zero-soliton sector. Thus we find

$$\begin{aligned}C^{(1)}(z,\tau)&=\langle\cos(\Phi_{Sol}(z,\tau)+\eta(z,\tau))\cos(\Phi_{Sol}(0,0)+\eta(0,0))\rangle^{(1)}\\&\approx\langle\cos(\Phi_{Sol}(z,\tau))\cos(\Phi_{Sol}(0,0))\rangle^{(1)}(1-\langle\eta^2(0,0)\rangle^{(0)})\\&\quad+\langle\sin(\Phi_{Sol}(z,\tau))\sin(\Phi_{Sol}(0,0))\rangle^{(1)}\langle\eta(z,\tau)\eta(0,0)\rangle^{(0)}.\end{aligned} \tag{29}$$

The transversal correlation function is calculated analogously.

The averages in the subspace of the collective coordinate are taken using standard path-integral techniques. The longitudinal correlation function takes the form

$$C_q^{(1)}(z,\tau)=\frac{1}{Z_{Sol}}\oint\mathcal{D}q(\tau)\mathrm{e}^{-A^{(1)}[q(\tau)]}\cos(\Phi_{sol}(z-q(\tau)))\cos(\Phi_{Sol}(-q(0))) \tag{30}$$

with $A^{(1)}[q(\tau)]$ being the action connected with the free-particle Lagrangian L_{Sol} in (22). In order to eliminate the dependence of Φ_{Sol} on the integration variable $q(\tau)$, we use the standard method of auxiliary variables. We then have

$$\begin{aligned}C_q^{(1)}(z,\tau)&=\int\mathrm{d}q_0\int\mathrm{d}q_1\int\frac{\mathrm{d}\kappa_0}{2\pi}\int\frac{\mathrm{d}\kappa_1}{2\pi}\mathrm{e}^{\mathrm{i}\kappa_0q_0}\mathrm{e}^{\mathrm{i}\kappa_1q_1}\cos(\Phi_{Sol}(z-q_1))\cos(\Phi_{Sol}(-q_0))\\&\quad\times\frac{1}{Z_{Sol}}\oint\mathcal{D}q(\tau)\exp(-A^{(1)}[q(\tau)]+\mathrm{i}\kappa_0q(0)+\mathrm{i}\kappa_1q(\tau)).\end{aligned} \tag{31}$$

The path integration with respect to $q(\tau)$ can now be carried out exactly, yielding

$$Z_{Sol}\frac{2\pi}{L_z}\delta(\kappa_0+\kappa_1)\exp\left(-\frac{\kappa_0^2}{4}\sigma(\tau)\right) \tag{32}$$

with

$$\sigma(\tau)=\frac{8}{M'_{Sol}}\sum_{n=1}^{\infty}\frac{1-\cos(\nu_n\tau)}{\beta\nu_n^2}=\frac{2\beta}{M'_{Sol}}\left(\frac{|\tau|}{\beta}-\frac{\tau^2}{\beta^2}\right) \tag{33}$$

where $\nu_n=2\pi n/\beta$ are the bosonic Matsubara frequencies.

Upon inserting the expression (16) for Φ_{Sol} and integrating over the auxiliary variables κ_0 and κ_1, we find for the longitudinal spin-correlation function

$$\begin{aligned}C_q^{(1)}(z,\tau)&=\frac{1}{L_z\sqrt{\pi\sigma(\tau)}}\int\mathrm{d}q_0\int\mathrm{d}q_1\exp\left(-\frac{(q_0-q_1)^2}{\sigma(\tau)}\right)\\&\quad\times\left(1-2\,\mathrm{sech}^2\left(\frac{\omega_0}{c}(z-q_1)\right)\right)\left(1-2\,\mathrm{sech}^2\left(\frac{\omega_0}{c}q_0\right)\right).\end{aligned} \tag{34}$$

It is straightforward to perform the Fourier transformation with respect to z. Then the longitudinal correlation function takes the form

$$C_q^{(1)}(k,\tau) = \left(1 - \frac{8c}{L_z\omega_0}\right)\delta(k) + \frac{2}{L_z}\exp\left(-\frac{\sigma(\tau)k^2}{4}\right)\left(\frac{\pi k c^2}{\omega_0^2\sinh(k\pi c/2\omega_0)}\right)^2. \tag{35}$$

In the transversal spin-correlation function $S_q^{(1)}$, the leading Bragg term is missing, and the hyperbolic sine has to be replaced by the hyperbolic cosine.

The Fourier transformation with respect to the imaginary time τ and subsequent analytical continuation ν_n to $-\mathrm{i}\omega$ results in the unique replacement

$$\exp\left(-\frac{\sigma(\tau)k^2}{4}\right) \to \frac{\sqrt{2\pi\beta M'_{Sol}}}{|k|}\exp\left(-\frac{\beta M'_{Sol}}{2k^2}\left(\omega - \frac{k^2}{2M'_{Sol}}\right)^2\right). \tag{36}$$

This form is similar to a former result obtained by Mikeska [13]. As regards difference, expression (36) also describes a shift of the maximum due to the possibility of thermal activation of the soliton.

4. The spectral density

The coupling functions (15) contain summations over the components of the spins involved. Since we are describing a magnetic system with strong in-plane anisotropy, the z-component of the spins is negligibly small, which is a condition for the validity of the sine–Gordon model. Taking the direction of the movement of the kinks parallel to z, we have only to deal with the cases $\mu\nu = xx, xy, yx, yy$ in $D_{\mu\nu}^{\lambda z}$.

To introduce the collective coordinate $q(\tau)$, we first write the components of the spin $S(z,\tau)$ as functions of the angular fields $\Phi(z,\tau)$ and $\theta(z,\tau)$. Then we set $\theta \equiv \pi/2$, and finally we insert the classical solutions (16) for $\Phi(z,\tau)$. Via this procedure, couplings between magnetic fluctuations and phonons are disregarded, as they are represented by fourth-order terms in the Lagrangian (1). We now have to distinguish between diagonal coupling functions ($\mu\nu = xx, yy$)

$$F_{xx} = -\frac{8S^2}{c}\frac{\omega_0}{c}D_{xx}\frac{\sinh(\chi)\{\cosh^2(\chi) - 2\}}{\cosh^5(\chi)} \tag{37}$$

($F_{yy}/D_{yy} = -F_{xx}/D_{xx}$), and nondiagonal coupling functions ($\mu\nu = xy, yx$)

$$F_{xy} = -\frac{2S^2}{c}\frac{\omega_0}{c}D_{xy}\frac{\{8 - 8\cosh^2(\chi) + \cosh^4(\chi)\}}{\cosh^5(\chi)} \tag{38}$$

($F_{yx}/D_{yx} = F_{xy}/D_{xy}$), where $\chi = \omega_0[z - q(\tau)]/c$. If there were antikinks instead of kinks (replacing χ by $-\chi$ in (37) and (38)) this would lead to a reverse sign in the diagonal coupling functions, and the same sign in the nondiagonal coupling functions. Clearly, the related spectral densities are the same, as they are quadratic in the couplings.

Inserting these forms of the coupling functions into equation (13), and interchanging the integration over the space with the sum over the phonon momenta, we obtain for the influence functional the expression

$$S_{inf}[\Phi] = -\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau' \frac{1}{2\pi\rho_{Res}}\sum_{k\lambda}\left\{\frac{\cosh(\Omega_{k\lambda}[\beta/2 - |\tau - \tau'|])}{8\Omega_{k\lambda}\sinh(\Omega_{k\lambda}\beta/2)} - \frac{1}{4\Omega_{k\lambda}^2}\delta(\tau - \tau')\right\}$$
$$\times \sum_{\alpha\beta} f_\alpha[q(\tau)]f_\beta[q(\tau')] \tag{39}$$

where we have introduced the coupling functionals $f_\alpha[q(\tau)]$ ($\alpha =$ d, n standing for diagonal and nondiagonal coupling respectively),

$$f_d[q(\tau)] = -\frac{8S^2}{c}\frac{\omega_0}{c}(D_{xx}-D_{yy})\int \mathrm{d}z\, \mathrm{e}^{\mathrm{i}k_z z}\frac{\sinh(\chi)\{\cosh^2(\chi)-2\}}{\cosh^5(\chi)}$$
$$= -\frac{8S^2}{c}\frac{c^2}{\omega_0^2}(D_{xx}-D_{yy})\frac{\mathrm{i}\pi k_z^2}{12\sinh(\pi c k_z/2\omega_0)}\left[2-\frac{c^2}{\omega_0^2}k_z^2\right]\mathrm{e}^{\mathrm{i}k_z q(\tau)} \tag{40}$$

$$f_n[q(\tau)] = -\frac{2S^2}{c}\frac{\omega_0}{c}(D_{xy}+D_{yx})\int \mathrm{d}z\, \mathrm{e}^{\mathrm{i}k_z z}\frac{8-8\cosh^2(\chi)+\cosh^4(\chi)}{\cosh^5(\chi)}$$
$$= -\frac{2S^2}{c}\frac{c^2}{\omega_0^2}(D_{xy}+D_{yx})\frac{\pi k_z^2}{3\cosh(\pi c k_z/2\omega_0)}\left[2-\frac{c^2}{\omega_0^2}k_z^2\right]\mathrm{e}^{\mathrm{i}k_z q(\tau)}. \tag{41}$$

There are three different combinations of the coupling functions. The first possibility is the combination of a diagonal with a nondiagonal function. This contribution vanishes for symmetry reaons. The second case is diagonal–diagonal (dd) coupling, in which two coupling functions of the type (40) are combined. Since we restrict ourselves to the nonrelativistic limit, it is sufficient to expand (39) in terms of $q(\tau)-q(\tau')$ up to second order. Replacing the sum over k_z by an integral over the phonons energy ω, we finally get

$$S_{inf}^{(dd)} \cong \tfrac{1}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau' K^{(dd)}(\tau-\tau')(q(\tau)-q(\tau'))^2 \tag{42}$$

with the damping kernel

$$K^{(dd)} = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\frac{\cosh(\omega[\beta/2-|\tau-\tau'|])}{\sinh(\omega\beta/2)}J^{(dd)}(\omega) \tag{43}$$

and where the spectral density of the coupling [7, 19] takes the form

$$J^{(dd)}(\omega) = J_0^{(dd)}\left(\frac{\omega}{\omega_1}\right)^5\left[\left(\frac{\omega}{\omega_1}\right)^2-2\right]^2\Big/\sinh^2\left(\frac{\pi}{2}\frac{\omega}{\omega_1}\right). \tag{44}$$

The constant $J_0^{(dd)}$ is related to the strength of the magnetoelastic interaction by

$$J_0^{(dd)} = \frac{\pi^2}{36}\frac{\omega_0}{c}\frac{S^4}{a^4 v_\lambda^2 \rho_{Res}}\{(D_{xx})^2-2D_{xx}D_{yy}+(D_{yy})^2\}. \tag{45}$$

Here we introduced the frequency $\omega_1 = v_\lambda\omega_0/c$ corresponding to a phonon wavelength which is of the order of the width of the kink. Below we shall see that ω_1 is the characteristic frequency of the dissipative process.

The third case is nondiagonal-nondiagonal (nn) coupling. The spectral density of the coupling resulting from similar calculations is

$$J^{(nn)}(\omega) = J_0^{(nn)}\left(\frac{\omega}{\omega_1}\right)^5\left[\left(\frac{\omega}{\omega_1}\right)^2-2\right]^2\Big/\cosh^2\left(\frac{\pi}{2}\frac{\omega}{\omega_1}\right) \tag{46}$$

with the coupling parameter

$$J_0^{(nn)} = \frac{\pi^2}{36}\frac{\omega_0}{c}\frac{S^4}{a^4 v_\lambda^2 \rho_{Res}}\{(D_{xy})^2+2D_{xy}D_{yx}+(D_{yx})^2\}. \tag{47}$$

The functions $J^{(dd)}(\omega)$ and $J^{(nn)}(\omega)$ are shown in figure 1.

The damping constants $J_0^{(dd)}$ and $J_0^{(nn)}$ have similar structure, and numerically they are of the same order of magnitude. If in addition the components of the magnetoelastic tensor

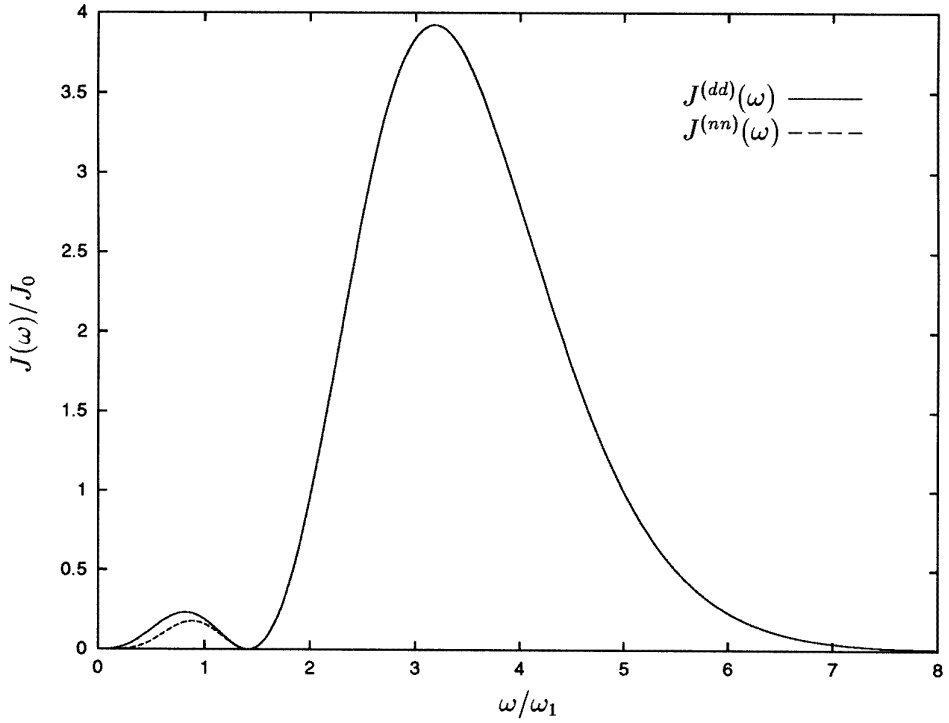

Figure 1. The spectral densities $J^{(dd)}(\omega)$, $J^{(nn)}(\omega)$ according to (44) and (46). The only differences between these densities appear far below $\omega \approx \pi\omega_1$ and are of minor importance.

$D_{\mu\nu}$ are similar in magnitude, the (dd) coupling is suppressed, but not the (nn) coupling. For tetragonal symmetry, we have $D_{xx} = D_{yy}$, so only the (nn) coupling is active.

The spectral densities $J^{(nn)}(\omega)$ and $J^{(dd)}(\omega)$ are zero at $\omega = \sqrt{2}\omega_1$. Hence, phonons with frequency within the frequency gap are decoupled from the kink and do not participate in the dissipation.

We find two peaks in the spectral densities. The major one is located near to $\omega_{max} \approx 3.2\omega_1 \approx \pi\omega_1$. Connected with this frequency there is a wave vector $k_{max} \approx \pi\omega_0/c$, and a typical length $l^* \approx 2c/\omega_0$, which is of the order of magnitude of the extension of a static kink. Phonons having energies around ω_{max} contribute most effectively to the dissipation.

A much smaller peak is found near $\omega = 0.8\omega_1$. The magnitude of this peak differs between $J^{(dd)}(\omega)$ and $J^{(nn)}(\omega)$. The typical length related to this frequency is $4l^* \approx 8c/\omega_0$.

Since the spectral densities decay as $J(\omega) \propto \omega^7 \exp(-\pi\omega/\omega_1)$ for large ω, there is no UV divergence when ω_1 is finite. Note that a Debye cut-off in the phonon dispersion relation, typically at $\omega = \omega_D = 2\pi v_\lambda/a$, has little effect, since this cut-off frequency is ususally much higher than the resonance frequancy $\pi\omega_{max}$, as, e.g., in $CsNiF_3$.

For small ω, the densities behave as $J^{(dd)}(\omega) \propto \omega^3$ and $J^{(nn)}(\omega) \propto \omega^5$, describing deep superohmic behaviour [19]. At this point it is interesting to note that Garg and Kim [4] found $J(\omega) \propto \omega^3$ for the single-domain particles.

An interesting result is that most of the damping comes from phonons with energies near ω_{max}. This may be used to design or select materials with special dissipative characteristics.

For instance, a material with high phonon density around this energy will exhibit quite large dissipative effects. On the other hand, if the phonon density is high within the gap near $\sqrt{2}\omega_1$, the material will show very weak dissipation.

5. Partition function and correlation functions in the presence of phonon damping

The forms (44) and (46) are far too complicated for performing calculations in analytic form. In a reasonable approximation, only phonons at the resonance frequency $\omega_{res} \approx \pi\omega_1$ may be taken into account. Matching the integrals over the spectral densities, we then have

$$J(\omega) \approx 9\omega_1 J_0 \delta(\omega - \pi\omega_1) \tag{48}$$

where the superscripts (dd) and (nn) indicating the coupling symmetry have been omitted. This approximation would be very rough if we were looking at the damping coefficient $\gamma(\omega)$ whose real part is related to the spectral density $J(\omega)$ by $\gamma'(\omega) \propto J(\omega)/\omega$. On the other hand, it gives reasonable approximation for the damping kernel, which is an integral over the spectral density:

$$\tilde{K}(\nu_n) = \frac{2}{\pi\beta}\int_0^\infty \mathrm{d}\omega \frac{\omega J(\omega)}{\omega^2 + \nu_n^2} \approx \frac{18J_0}{\beta}\frac{\omega_1^2}{\nu_n^2 + \pi^2\omega_1^2}. \tag{49}$$

The full kernel is compared with the approximate one in figure 2.

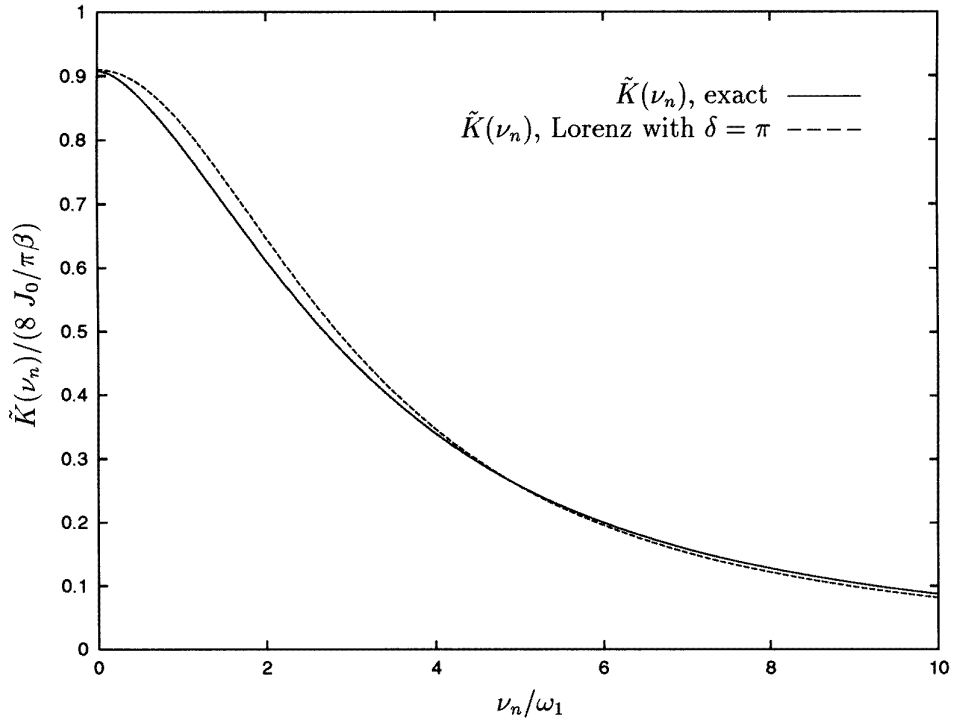

Figure 2. The Fourier transform of the damping kernel $\tilde{K}(\nu_n)$ with the spectral density (44) and with the approximation (48).

The partition function is modified due to the influence functional [7]

$$A_{inf}[q(\tau)] = \tfrac{1}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau' \, K(\tau-\tau')(q(\tau)-q(\tau'))^2 = \beta^2 \sum_{n=-\infty}^{\infty} |\tilde{q}_n|^2(\tilde{K}(0)-\tilde{K}(\nu_n)). \tag{50}$$

As (50) is quadratic in q (but nonlocal in imaginary time), the partition function may be calculated without difficulty. We find that the form (24) is changed into

$$Z_{Sol} = \mathrm{e}^{-M'_{Sol}c^2\beta}\sqrt{\frac{M'_{Sol}}{2\pi\beta}}\,\frac{\omega_2}{\omega_1}\,\frac{\sinh(\pi\omega_1\beta/2)}{\sinh(\pi\omega_2\beta/2)} \tag{51}$$

where

$$\omega_2 = \omega_1\sqrt{1+\frac{36J_0}{M'_{Sol}\omega_1^2\pi^4}}.$$

Figure 3 shows that the approximation (48) leads to results which are in fairly good agreement with the exact numerical results for the partition function for different coupling strengths $C = 36J_0/M'_{Sol}\omega_1^2\pi^4$. The approximation gets less appropriate as the temperature is lowered. This is because (48) does not describe correctly the low-frequency behaviour of the spectral density.

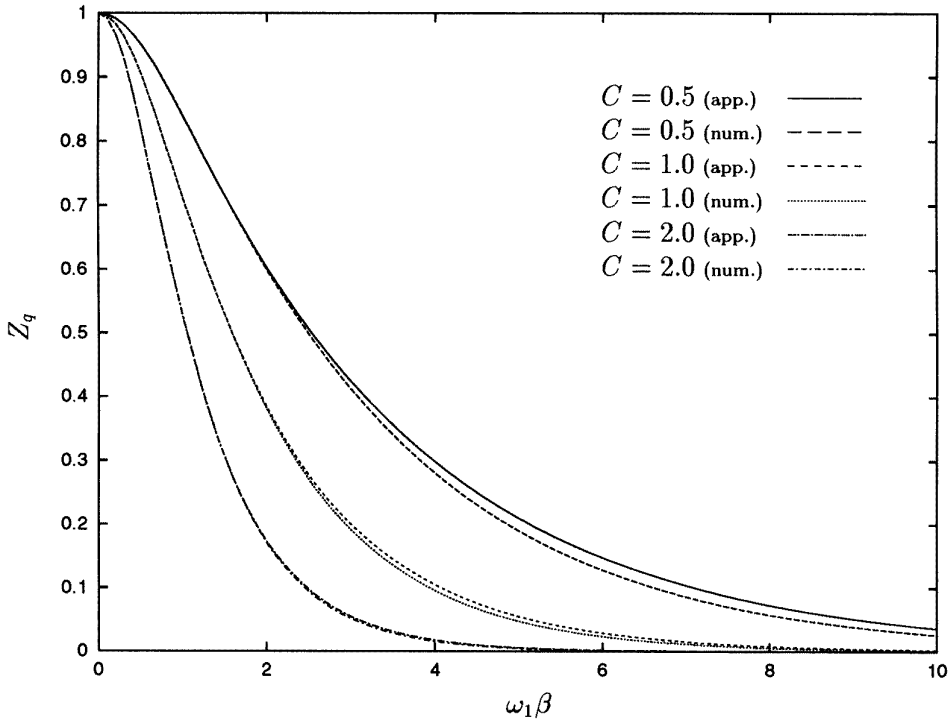

Figure 3. The partition function (51) for different coupling strengths $C = 36J_0/M'_{Sol}\omega_1^2\pi^4$ in comparison with exact numerical results.

The correlation functions are modified similarly. The only place where the imaginary time τ appears in $C_q^{(1)}(z,\tau)$ is in the function $\sigma(\tau)$ given in (33). This quantity now takes the form

$$\tilde{\sigma}(\tau) = 2\beta M'_{Sol}\bar{\alpha}_D\left(\frac{|\tau|}{\beta} - \frac{\tau^2}{\beta^2}\right) + \frac{2\alpha_D}{\omega_2 M'_{Sol}}\left(\coth\left(\frac{\omega_1\beta}{2}\right) - \frac{\cosh(\omega_3[\beta/2 - |\tau|])}{\sinh(\omega_3\beta/2)}\right) \tag{52}$$

where $\omega_3 = \pi\omega_2$, and $\alpha_D = 1 - \omega_1^2/\omega_2^2$ and $\bar{\alpha}_D = \omega_1^2/\omega_2^2$ are the damping parameters.

From (36) we see how to perform the analytical continuation of an exponential function containing terms $\propto \tau$ and $\propto \tau^2$. Therefore, we expand $\exp(-\tilde{\sigma}(\tau)k^2/4)$ with (52) into a sum of exponentials and perform the analytical continuation for each term separately. The result is

$$\begin{aligned}\exp\left(-\frac{\tilde{\sigma}(\tau)k^2}{4}\right) \to \sqrt{\frac{2\pi\beta M'_{Sol}}{\bar{\alpha}_D k^2}}\exp\left(-\frac{\alpha_D k^2}{2\omega_3 M'_{Sol}}\coth\left(\frac{\omega_3\beta}{2}\right)\right) \\ \times \sum_{l=-\infty}^{\infty} I_{|l|}\left(\frac{\alpha_D k^2}{4\omega_3 M'_{Sol}\sinh(\omega_3\beta/2)}\right)\mathrm{e}^{l\omega_3\beta/2} \\ \times \exp\left(-\frac{\beta M'_{Sol}}{2\bar{\alpha}_D k^2}\left(\omega - l\omega_3 - \frac{\bar{\alpha}_D k^2}{2M'_{Sol}}\right)^2\right)\end{aligned} \tag{53}$$

where $I_l(z)$ is a modified Bessel function.

The interpretation of this result is as follows. The expression (53) contains a major peak at $l = 0$ which by far dominates over all the other peaks. The distance between neighbouring peaks is $\omega_3 = \pi\omega_1/\sqrt{\bar{\alpha}_D}$, which is increasing with increasing dissipation strength. The additional peaks in the scattering function are signatures of the interaction of the soliton with the phonon bath. Because of the interactions, a neutron may be scattered by the soliton thereby emitting or absorbing a phonon simultaneously. Similar peaks have been predicted in an isotropic Heisenberg system by Fivez and De Raedt [20] using the Mori formalism.

The change of the central peak indicates that the soliton mass is increased according to $M_{eff} = M'_{Sol}/(1 - \alpha_D)$. The low-frequency part of the spectral density influences the dynamics of the soliton at long times. On the other hand, the high-frequency part gives rise to mass renormalization. The change of the mass is given by [19]

$$\Delta M_{hf} = \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}\omega}{\omega^3} J_{hf}(\omega) = \frac{M'_{Sol}\alpha_D}{2(1-\alpha_D)}. \tag{54}$$

A realistic estimate [4, 21] of the relevant components of the magnetoelastic tensor D_{mn}^{zz} gives $\alpha_D \approx 0.06$. Hence the phonon damping is very weak. The change of the transversal spin-correlation function for this value of α_D is shown in figure 4.

6. Concluding remarks

In this paper, we have suggested a mechanism for the coupling of a solitary solution of the sine–Gordon equation to an environment of phonons. Upon employing the method of collective coordinates, we have mapped the original problem onto that of a free particle moving under the influence of friction. The related spectral density of the coupling has been calculated. It shows a pronounced maximum at a resonance frequency which corresponds to phonons with a wavelength comparable to the spatial extent of the soliton.

Taking into account the resulting form of the spectral density, we have calculated the longitudinal and transversal spin-correlation functions. We have found that the coupling to

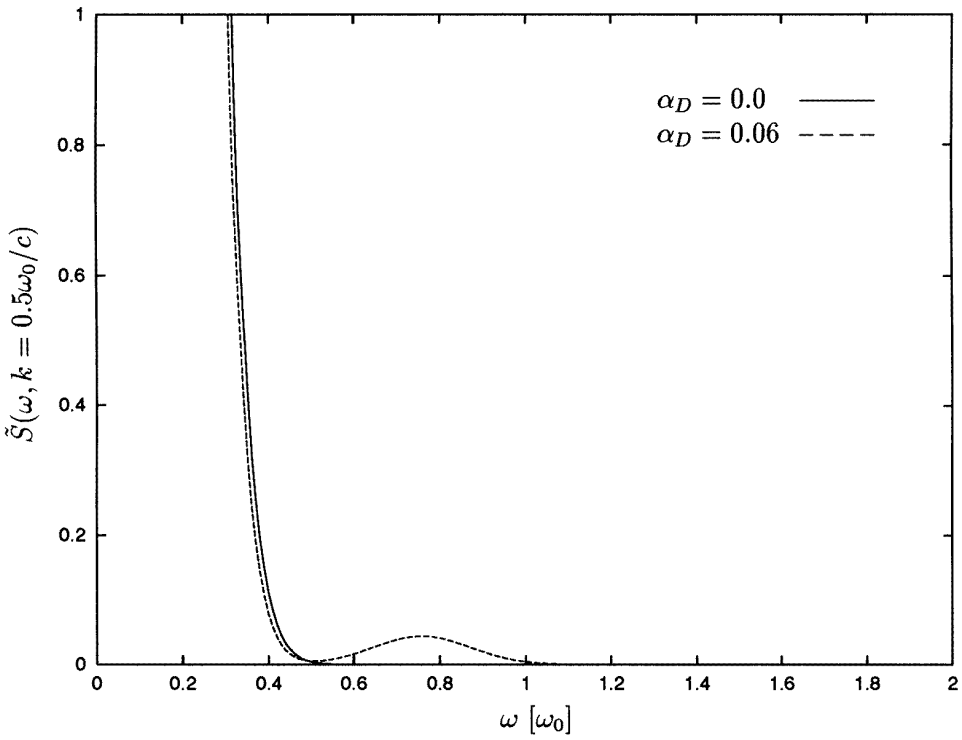

Figure 4. The transversal spin-correlation function $\tilde{S}(\omega, k)$ at $k = -0.9\omega_0/c$ with the realistic strength $\alpha_D = 0.06$. Only the lower part of the major peak at around $\omega = 0$ appears.

lattice vibrations gives rise to additional peaks in the structure factor for neutron scattering. The separation of these peaks depends essentially on the resonance frequency, but increases with increasing damping. The relative intensity of such peaks crucially depends on the damping strength. A possible material for which to study spin relaxation in quasi-one-dimensional systems by neutron scattering is $CsNiF_3$. As the material is insulating, the dominant dissipative effects are due to the coupling to lattice vibrations, and the theoretical study presented here should apply.

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